

# Negative magnetoresistance and phase slip process in superconducting nanowires

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We argue that the negative magnetoresistance of superconducting nanowires, which was observed in recent experiments, can be explained by the influence of the external magnetic field on the critical current of the phase slip process. We show that the suppression of the order parameter in the bulk superconductors made by an external magnetic field can lead to an enhancement of both the first  $I_{c1}$  and the second  $I_{c2}$  critical currents of the phase slip process in nanowires. Another mechanism of an enhancement of  $I_{c1}$  can come from decreasing the decay length of the charge imbalance  $\lambda_Q$  at weak magnetic fields because  $I_{c1}$  is inversely proportional to  $\lambda_Q$ . The enhancement of the first critical current leads to a larger intrinsic dissipation of the phase slip process. It suppresses the rate of both the thermo-activated and/or quantum fluctuated phase slips and results in decreasing the fluctuated resistance.

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## I. INTRODUCTION

Recently several experimental groups have observed a negative magnetoresistance (NMR) of the superconducting nanowires at the temperature lower than the critical temperature [1, 2, 3, 4, 5, 6]. In Ref. [3, 6] the authors have directly demonstrated that in their case the effect is connected with the suppression of the superconductivity in the bulk superconductors caused by the applied magnetic field (it was called as 'anti-proximity effect'). They have also found an enhancement of the critical current of the nanowires when the magnetic field turns bulk superconductors to the normal state. This result convinced us that the observed NMR could be connected with the increase in the critical currents of the phase slip process. To illustrate how the change of the critical current value can influence the fluctuated resistance we use a well-known model of the point-like Josephson junction with a finite capacitance.

The current-voltage (IV) characteristics of such a junction is hysteretic and the parameter which governs the IV characteristics is the ratio between the effective "mass" (which is proportional to the capacitance  $C$  of the junction) and the parameter describing the effect of intrinsic dissipation (which is inversely proportional to the resistance  $R$  of the junction) [7]. In the theory of Josephson junctions this ratio is called the damping parameter  $\beta_c = 2eI_c R^2 C / \hbar$  ( $I_c$  is the critical current of the Josephson junction with zero capacitance). The larger  $\beta_c$  is the larger is the hysteresis and the smaller current  $I_r$  at which the voltage vanishes in the junction (see Fig. 1). For small values of  $\beta_c$  the current  $I_r$  is practically equal to  $I_c$  and the hysteresis is absent. The effect of fluctuations leads to the appearance of the finite resistance at  $I < I_r$  and the absence of the hysteresis (phase slip process does not stop at  $I_r < I < I_c$  once it is launched by a

fluctuation) [7]. If we increase the value of the current  $I_r$  (by decreasing the resistance of the junction) the fluctuated resistance at fixed  $I \ll I_r$  decreases (compare gray and black dashed curves in Fig. 1). It is the consequence of the general rule, that the increase in the intrinsic dissipation  $W$  in the system (in case of Josephson junction  $W = V^2/R$ ) suppresses both quantum [8] and thermo-activated fluctuations [9].

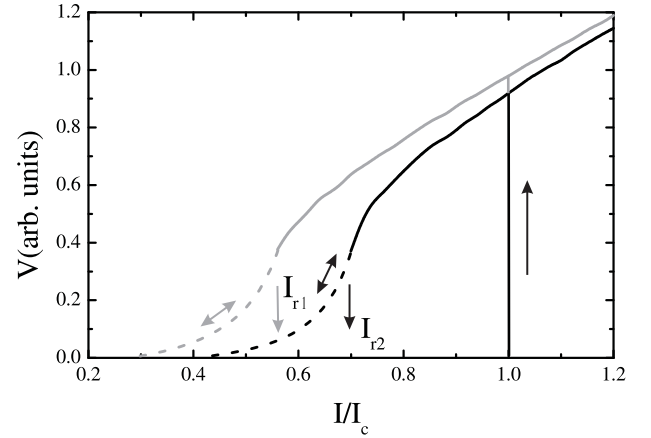


FIG. 1: Current-voltage characteristics of Josephson junction with a finite capacitance and  $I_{r1} < I_{r2}$  (parameter  $\beta_c = 5$  and 3 respectively) in the case of zero fluctuations (solid curves). Dashed curves show schematically a non-zero voltage response at  $I < I_r$  due to strong fluctuations. In the latter case the hysteresis in current-voltage characteristics disappears.

The aim of the present paper is to show that the magnetic field can enhance the critical currents of the phase slip process and hence intensifies the intrinsic dissipation. As in the case of Josephson junction it leads to the suppression of the rate of fluctuations and the decrease of the fluctuated resistance.

However, there is a difference between the results in Refs.[1, 2, 4, 5] and Refs.[3, 6]. In Refs. [1, 2, 4, 5] the resistance of superconducting nanowires monotonically decreases with the increase of  $H$  (up to some critical

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value) while in Refs. [3, 6] the resistance is almost constant at weak magnetic fields and drops suddenly near the critical field of the bulk superconductors. Therefore, we conclude that the results are connected with different mechanisms of the enhancement of the critical currents in the PS process.

Before considering those mechanisms let us discuss the physical meaning of the first and the second critical currents of the phase slip process in superconducting wires.

The paper is organized as follows. In section II we observe the critical currents of the phase slip process in quasi-1D superconductors. In section III we study the influence of the magnetic field on those critical currents. Finally, in section IV we discuss our results and make a comparison with the experiments and other theoretical works.

## II. CRITICAL CURRENTS OF THE PHASE SLIP PROCESS

It has been known for a long time that the phase slip process in quasi 1D superconducting wires [10] is a hysteretic process (see for example review [11] and books [7, 12]). If we start from the superconducting state and gradually increase the applied current the superconducting state becomes unstable at the current  $I_{c2}$  (it is analog of the critical current of Josephson junction  $I_c$ ) which is equal to the product of the depairing current density  $j_{dep}$  (for wire without defects) on the square of the cross-section of the sample  $S$ . The periodic in time oscillations of the order parameter in one or several points along the superconductor destroy the zero-resistance state and bring a finite resistance (less than normal one) to the system. This process is called a phase slip (PS) process and the points are called as phase slip centers (PSC) [7, 11, 12]. If we decrease the current below  $I_{c2}$ , the phase slip process can vanish at the current  $I_{c1} < I_{c2}$  (which roughly corresponds to the current  $I_r$  of Josephson junction).

The physical origin of the current  $I_{c1}$  was clarified in Ref. [13] using the extended quasi-1D time-dependent Ginzburg-Landau equations [14, 15]. It has been found that there are two characteristic times which govern the dynamic of the order parameter in the phase slip center. These are the time relaxation of the absolute value of the order parameter  $\tau_{|\psi|}$  and the relaxation time of the phase gradient  $\tau_{\nabla\phi}$  (which is proportional to the momentum of superconducting electrons).

First characteristic time can be estimated from the time-dependent Ginzburg-Landau equation for dynamics of  $|\Delta|$

$$\tau_{GL}u\sqrt{1+\gamma^2|\Delta|^2}\frac{\partial|\Delta|}{\partial t} = \xi^2\frac{\partial^2|\Delta|}{\partial s^2} + |\Delta|(1-|\Delta|^2-(\xi\nabla\phi)^2), \quad (1)$$

where  $\Delta = |\Delta|e^{i\phi}$  is the order parameter in Ginzburg-Landau equations normalized to its equilibrium value at a specific temperature  $|\Delta|_{eq} = 4k_B T_c u^{1/2}(1 - T/T_c)^{1/2}/\pi$ ,

$\tau_{GL} = \hbar/(k_B(T_c - T)u)$  is the Ginzburg-Landau relaxation time,  $\xi = (8k_B(T_c - T)/\pi\hbar D)^{-1/2}$  is a coherence length ( $D$  is a diffusion coefficient),  $\gamma(T) = 2|\Delta|_{eq}(T)\tau_E/\hbar$  is the parameter in time-dependent GL equations,  $\tau_E$  is the energy relaxation time for electrons near Fermi level and  $u \simeq 5.79$  is a number [14, 15]. Numerical analysis shows [13] that the amplitude of oscillations of  $|\Delta|$  in phase slip center is decreasing with the increase of  $\gamma$  and it is normally much smaller than  $|\Delta|_{eq}$ . It allows to neglect the nonlinear term  $|\Delta|^3$  in the right hand side (RHS) of the Eq. (1) near the core of the phase slip center and it immediately gives us  $\tau_{|\Delta|} \simeq u\tau_{GL}$ . We can also identify  $\tau_{|\Delta|}$  as a relaxation time of the longitudinal mode in the superconductors  $\tau_{|\Delta|} \simeq \tau_E k_B T_c / \Delta$  [7, 16].

The second characteristic time  $\tau_{\nabla\phi}$  for long wires  $L \gg \lambda_Q$  ( $\lambda_Q(\gamma \gg 1) \simeq \sqrt{\gamma/u}\xi \gg \xi$  is the decay length of the charge imbalance [7]) could be estimated by using the Ginzburg-Landau equation for dynamics of the phase of the order parameter [13]

$$\frac{\hbar}{2e}\frac{\partial\phi}{\partial t} = -\varphi + \lambda_Q^2\frac{\partial^2\varphi}{\partial x^2}. \quad (2)$$

For  $\lambda_Q \gg \xi$  or  $\gamma \gg 1$  the order parameter *mainly* oscillates in a small region (which decreases while increasing  $\gamma$ ) around the phase slip center with the size smaller than  $\xi$ . Therefore, we need to estimate  $\tau_{\nabla\phi}$  in this area. As a result we have  $\tau_{\nabla\phi} \sim \tau_{GL}I_0\xi/I\lambda_Q$  ( $I$  is an applied current,  $I_0 = S\hbar/2e\tau_{GL}\xi\rho_n$  is proportional to the depairing current ( $I_{c2} = \sqrt{4/27}I_0$ ),  $\rho_n$  is a normal state resistivity and we take into account that  $-\partial\varphi/\partial x(x=0) = I_n(x=0)\rho_n/S \sim I\rho_n/S$ ).

When  $\tau_{|\Delta|} \lesssim \tau_{\nabla\phi}$  the phase slip process is impossible as a periodic one in the time oscillating process [13] at  $I < I_{c2}$ . It allows us to estimate the first critical current of long  $L \gg \lambda_Q$  wires

$$I_{c1} \sim \frac{I_0\tau_{GL}\xi}{\tau_{|\Delta|}\lambda_Q} = \frac{\hbar}{2e\tau_{\Delta}} \frac{S}{\rho_n\lambda_Q} = \frac{\hbar}{e\tau_{\Delta}} \frac{1}{R_{PS}}, \quad (3)$$

where  $R_{PS} = 2\lambda_Q\rho_n/S$  may be called as a resistance of the phase slip process [17]. Note, that in case of Josephson junction current  $I_r$  is also inversely proportional to the intrinsic resistance [7].

Due to the above threshold condition there is a voltage jump  $\Delta V \simeq 1/\tau_{|\Delta|}$  at  $I = I_{c1} < I_{c2}$  [13]. If current  $I_{c1}$  defined by the above expression becomes larger than  $I_{c2}$  (at  $\lambda_Q \lesssim \xi$  or/and  $\tau_{|\Delta|} \lesssim \tau_{GL}$ ) then the voltage gradually increases from zero at  $I = I_{c2}$  and  $\Delta V = 0$ . In this limit our estimations for  $\tau_{|\Delta|}$  and  $\tau_{\nabla\phi}$  become invalid.

From Eq. (2) it follows that the superconducting electrons with momentum  $p \sim \nabla\phi$  being accelerated by the gradient of the electrochemical potential  $\mu_{e-ch} = e\varphi + \mu_{ch}$  where  $\mu_{ch}$  might be called as a chemical potential of the superconducting electrons. In Ref. [16] it is shown that  $-\mu_{ch} = e\varphi - \mu_{e-ch}$  is also proportional to the charge imbalance  $Q$  between hole-like and electron-like branches of the quasiparticle spectrum in superconductors. We may average Eq. (2) over the period of

oscillations of  $|\Delta|$  and in the case of two-dimensional geometry we have

$$\lambda_Q^2 \Delta Q - Q = 0. \quad (4)$$

When the width ( $w$ ) of the superconducting wire (which is connected to bulk superconducting reservoirs) is much less than  $\lambda_Q$  we can leave only the term with the Laplacian in Eq. (4) near the ends of the wire and solve 2D Laplace equation. Besides we can neglect the variation of  $Q$  over the width of the nanowire and solve 1D variant of Eq. (4) in the wire. As a result we obtain the charge imbalance at the ends of the nanowire  $Q_0 \simeq wQ_c/(\lambda_Q \sinh(L/2\lambda_Q))$  ( $Q_c$  is the charge imbalance in the phase slip center). Usually  $w/L \ll 1$  [3, 6] and  $Q_0 \ll Q_c$  even for short wires  $\lambda_Q \gg L$ . Therefore instead of 2D Eq. (4) we may use (in the wire) 1D equation

$$\lambda_Q^2 \frac{d^2 Q}{dx^2} - Q = 0, \quad (5)$$

with boundary conditions  $Q(\pm L/2) = 0$ ,  $Q(\pm 0) = \pm Q_c$ .

Using Eq. (5) with above boundary conditions it can be found that the current  $I_{c1}$  depends on the length of the nanowire. If the wire is much longer than the coherence length we may expect that the order parameter distribution in the core of PSC is not influenced by the bulk superconductors. Then the dynamics of the  $|\Delta|$  stays the same as for an infinite wire and both  $\tau_{|\Delta|}$  and  $\Delta V$  do not suffer any change. From the solution of Eq. (5) it might be easily seen that the normal current in the phase slip center (which is proportional to the applied one) grows with the decrease of  $L$  as  $I_n(0) \sim -dQ/dx(x=0) \sim 1/\tanh(L/2\lambda_Q)$  to provide the same charge imbalance  $Q_c \sim \Delta V/2 \sim 1/\tau_{|\Delta|}$  near the PSC. Therefore, the shorter the wire is the larger is  $I_{c1}$  ( $I_{c1} \sim 1/(\lambda_Q \tanh(L/2\lambda_Q))$ ) and for sufficiently short wires it becomes equal to  $I_{c2}$ . We should note that  $I_{c2}$  does not vary while the wire is much longer than  $\xi$ . Therefore, we expect that for short wires  $L \ll \lambda_Q$  the hysteresis in current voltage characteristics disappears [13].

### III. EFFECT OF THE MAGNETIC FIELD

#### A. First mechanism

Let us now discuss how an external magnetic field may influence  $I_{c1}$  and  $I_{c2}$ . First mechanism comes from the suppression of the order parameter in the bulk superconductors. In Fig. 2 we draw the qualitative distribution of  $|\Delta|$  and  $Q$  at  $H = 0$ ,  $H \lesssim H_c^{bulk}$  and  $H \gtrsim H_c^{bulk}$ . When drawing the curves we have assumed that the NS boundary forms far from the ends of the wire at  $H \lesssim H_c^{bulk}$  and approaches the wire at  $H \gtrsim H_c^{bulk}$ .

Due to conversing the normal current into the superconducting one at the NS boundary an additional charge imbalance appears at the ends of the wire and

an effective boundary condition for Eq. (5) becomes  $Q(\pm L/2) = \pm Q_0$ . It brings us the following expression for the first critical current

$$\frac{I_{c1}(H, L)}{I_{c1}(H=0, L=\infty)} = \frac{-Q_0/Q_c + \cosh(L/2\lambda_Q)}{\sinh(L/2\lambda_Q)} \quad (6)$$

Current  $I_{c1}$  increases with the growth of  $H$  because  $Q_0$  changes from zero to the maximal value (with the sign opposite to  $Q_c$  - see Fig. 2) when the NS boundary touches the end of the nanowire (in the latter case the expression for  $I_{c1}$  was found in Ref. [13]). With the increase of a magnetic field the point where  $Q = 0$  approaches the center of the wire (see Fig. 2). Hence we can say that the appearance of NS boundaries effectively shortens the superconductor (in sense that 'space' for phase slip process decreases) and it is the reason for an enhancement of  $I_{c1}$ .

Because the normal current exists on the finite distance from the NS boundaries inside the superconductor the current  $I_{c2}$  is enhanced too (for wires  $L \gg \xi$ ). Indeed, when the normal current penetrates far into the sample it decreases the superconducting component of the current because  $I_s + I_n = I$ . Hence, we need a larger applied current  $I$  to satisfy the condition  $I_{c2} = I_s$ .

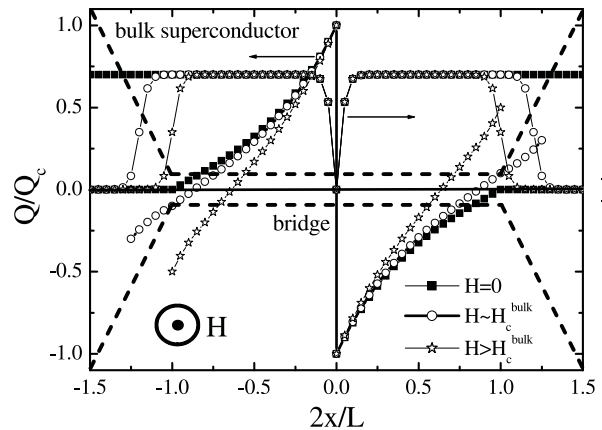


FIG. 2: Schematic distribution of the charge imbalance and order parameter in the superconducting bridge ( $L = 4\lambda_Q$ ) with one phase slip center at different values of the magnetic field.

The characteristic length of the discussed mechanism is the decay length of the charge imbalance. It means that the effect exists only in relatively short wires  $L \lesssim \lambda_Q$ . Besides the nanowire should not be wide, otherwise the critical field of the wire  $H_c \sim 1/(\xi w)$  and a bulk superconductor  $H_c^{bulk} \sim 1/(\xi^2)$  become close to each other and the magnetic field strongly suppresses  $|\Delta|$  in the wire (in framework of GL model  $|\Delta| = (1 - (H/H_c)^2)^{1/2}$ ). It leads to increasing  $\tau_{|\Delta|} \sim 1/(1 - (H/H_c)^2)$  and  $\lambda_Q \sim 1/|\Delta|^{1/2}$  [13] and hence to decreasing  $I_{c1}$  if the effect of the NS boundaries is weak. In derivation of Eq. (6) we suppose that both  $\tau_{|\Delta|}$  and  $\lambda_Q$  do not depend on the magnetic field. It is true if  $H \ll H_c$  and at some additional conditions (see the subsection below).

## B. Second mechanism

The second mechanism of a variation of  $I_{c1}$  comes from the dependence of pair-breaking mechanisms on the magnetic field due to an orbital effect [16, 18]. In Ref. [16] the decreasing  $\lambda_Q$  with the increase of the applied magnetic field was predicted for weak magnetic fields. Because the first critical current depends on  $\lambda_Q$  as  $I_{c1} \sim 1/\lambda_Q$  we can expect that  $I_{c1}$  increases in weak magnetic fields. Note that in contrast to the first mechanism  $I_{c2}$  decreases in this case.

The quantitative expression for  $\lambda_Q(H)$  was found in Ref. [16]

$$\lambda_Q(H) = \frac{4Dk_B T \tau_E}{\pi|\Delta|(H)} \left( \frac{1}{1 + \gamma(0)(H/H_c(0))^2} + \frac{1}{\gamma(T)^2} \right) \quad (7)$$

for temperatures close to  $T_c$ . The physical reason for the dependence of  $\lambda_Q$  on  $H$  is the following. The decay of the charge imbalance in superconductors occurs due to Andreev reflection process (for quasiparticles with energy less than  $|\Delta|$  near the NS boundary) or/and due to an inelastic electron-phonon interaction (for quasiparticles with the energy larger than  $|\Delta|$  and along the whole superconductor). A weak magnetic field almost does not influence the order parameter, but it smears the density of states of the quasiparticles [19] and makes possible Andreev reflection process of the quasiparticles with the energy larger than  $|\Delta|$ . It provides a faster relaxation of the charge imbalance and decreases  $\lambda_Q$ . A high magnetic field strongly suppresses the order parameter and makes the contribution to Andreev reflections smaller. It increases effective  $\lambda_Q$ .

We study this effect in the framework of the time-dependent GL equations. We use the field and temperature dependent parameter  $\gamma_H = \gamma/(1 + \gamma(H/H_c)^2(1 - T/T_c)^{1/2})^{1/2}$  in Eq. (1,2) and add the term  $(-|\Delta|(H/H_c)^2)$  in the right hand side of Eq. (1) [13] (we use dependence  $H_c(T) = H_c(0)\sqrt{1 - T/T_c}$ ). For example for tin ( $\gamma \simeq 100(1 - T/T_c)^{1/2}$ ) the effect is expected to be weak and for aluminium ( $\gamma \simeq 10^4(1 - T/T_c)^{1/2}$ ) it should be noticeable already at the temperature close to  $T_c$ .

In our calculations we take  $T = 0.9T_c$  and several values of the parameter  $\gamma$  (see Fig. 3(a)). At a weak magnetic field  $I_{c1}$  increases if the parameter  $\gamma$  is sufficiently high. It occurs not only due to decreasing  $\lambda_Q \sim \sqrt{\gamma_H}$  but also due to decreasing  $\tau_{|\Delta|} \sim \gamma_H$ . The large magnetic field suppresses the order parameter considerably and it leads to decreasing  $I_{c1}$  and  $I_{c2}$  (see the end of Sec. III A). Because for short wires  $I_{c1}$  does not depend on  $\lambda_Q$  the enhancement of the  $I_{c1}$  due to the discussed mechanism should be weakened in such a samples. The numerical calculations confirmed that statement (see Fig. 3(b)).

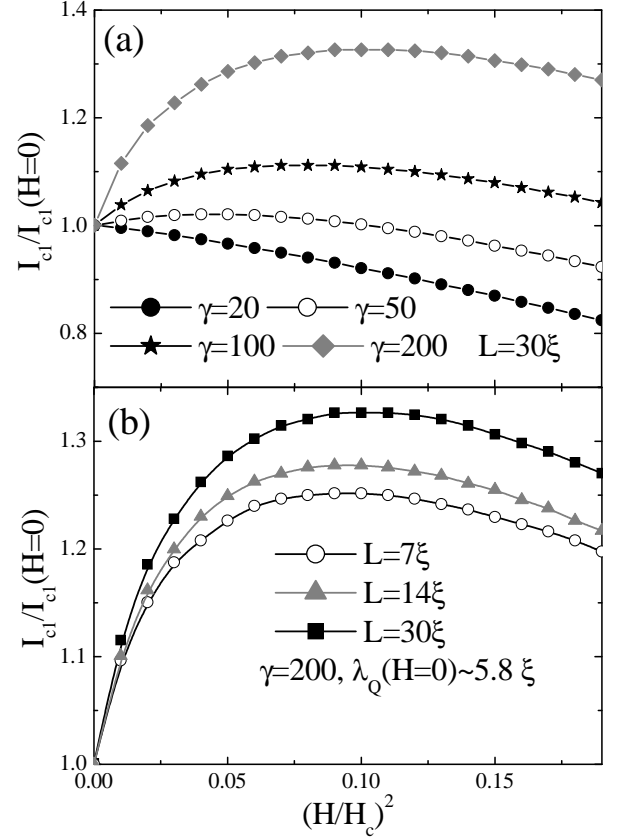


FIG. 3: (a) Dependence of  $I_{c1}$  on the magnetic field which was found from the numerical solution of Eqs. (1,2) with a field dependent parameter  $\gamma_H = \gamma/(1 + \gamma(H/H_c)^2(1 - T/T_c)^{1/2})^{1/2}$  at different values of  $\gamma$ . (b) Dependence of  $I_{c1}$  on the magnetic field at the fixed value of  $\gamma = 200$  and different lengths of the superconducting wire.

## IV. DISCUSSION

We can present simple interpretations of our results. In long ( $L \gg L_Q$ ) wires the resistance of the phase slip process  $R_{PS}$  is proportional to  $2\lambda_Q\rho_n/S$  [17]. In short ( $L \lesssim L_Q$ ) wires  $R_{PS}$  is proportional to the length of the wire  $R_{PS} \simeq L\rho_n/S$  because the electric field is close to zero in bulk superconductors. When a short wire is bounded by the normal metal the resistance of the phase slip process *decreases* by the value which is proportional to the resistance of the NS boundaries  $R_{PS} \simeq L\rho_n/S - 2R_{NS}$ . When one applies a magnetic field  $R_{PS}$  can decrease due to decreasing  $\lambda_Q$  (in long wires) or due to the appearance of  $R_{NS}$  (in short wires) and as in the case of Josephson junction current  $I_{c1} \sim \Delta V/R_{PS}$  increases. The intrinsic dissipation  $W \sim V^2/R_{PS}$  increases too and it suppresses the fluctuations of the phase of the order parameter and results in decreasing the fluctuated resistance.

The extended time-dependent Ginzburg-Landau equations are valid for a narrow temperature interval close to the critical temperature  $|T_c - T| < \hbar/k_B T_E$  where

$\xi(T) > L_E = \sqrt{D\tau_E}$ . However, we may suppose that traces of the high temperature dynamics should exist at low temperatures too. It was experimentally found for a tin nanowire with  $L = 6\mu m$  the presence of several phase slip centers (see Fig. 3b in Ref. [20]) at  $T = 0.5K \simeq 0.12T_c$ . First phase slip brings a finite resistance which is equivalent to the resistance of the piece of the wire with the length about  $1.5\mu m$  and hence using SBT theory [17] we obtain  $\lambda_Q(Sn) \simeq 750nm$ . It is much larger than the coherence length in tin at this temperature ( $\xi(Sn, T=0) \lesssim 55nm$ ).

In Ref. [21] the S-behavior of the current voltage characteristics of superconducting nanowires was found in a voltage driven regime at low temperatures. Time-dependent Ginzburg-Landau equations with a large value of  $\gamma$  give qualitatively the same result [13, 21]. The voltage jump  $\Delta V$  was extracted from the experimental data and showed a qualitative agreement with the theoretical temperature dependence  $\Delta V(T)$  [13].

In the recent paper [22] the pronounced hysteresis of the IV characteristic of the Pb nanowire with several phase slip centers was observed at a low temperature. In the preceding experiments [13, 21] the hysteresis was hidden by a strong external noise. Proper filtering suppressed the noise [22] and revealed the hysteretic behavior with well identified currents  $I_{c1}$  and  $I_{c2}$ .

The above experiments support the idea that the low-temperature properties of the phase slip process resemble such ones at high temperatures. Therefore, we expect that our results based on the numerical solution of Eqs. (1,2) and Eqs. (4,5) are applicable for *qualitative* analysis of the phase slip process at low temperatures.

The magnetic field dependence of the first and second mechanisms of the current enhancement are rather different. In the case of the first mechanism the enhancement occurs at the moment when bulk superconductors switch to the normal state. In the second mechanism the current  $I_{c1}$  increases smoothly with  $H$  (see Fig. 3). Therefore, we believe that the first mechanism may be responsible for the effects observed in Refs.[3, 6] while the second mechanism is connected with the experiments in Refs. [1, 2, 4, 5].

Indeed, the theory for the first mechanism clarifies why the 'anti-proximity effect' [3, 6] is weakened in long Zn nanowires with  $L = 30\mu m$  ( $\lambda_Q(Zn) \sim 22\mu m$ ) and is absent in Sn nanowires with lengths  $L = 6 - 30\mu m$  ( $\lambda_Q(Sn) \sim 750nm$  - see our estimation above). They seem to be too long to observe the 'anti-proximity effect'.

In Ref. [6] direct measurements demonstrated an increase of the critical current in the superconducting nanowires. We identify this current as the first critical current of the phase slip process. The second critical current was not observed in the experiment due to a high rate of fluctuated PSC which reveals itself in the finite resistance of the wire at  $I \rightarrow 0$ . The absence of hysteresis is also typical for Josephson junction with a high rate of fluctuations [7].

To observe the first mechanism of NMR (due to cur-

rent enhancement) the width  $w$  or diameter  $d$  of the wire should be relatively small to provide a condition  $H_c \sim 1/(w, d) \gg H_c^{bulk}$ . Otherwise the magnetic field  $H = H_c^{bulk}$  strongly suppresses the order parameter in the wire. It leads to decreasing both critical currents  $I_{c1}$ ,  $I_{c2}$  (see and of Sec. III A) and results in a negligible effect of the normal boundaries. We believe that it is the reason for the observed dependence of the 'anti-proximity effect' on the width of the nanowires in Ref. [3]. The authors found no effect for a wire with  $d = 70nm \sim \xi/2$  and pronounced effect for a nanowire with  $d = 40 \sim \xi/4$ .

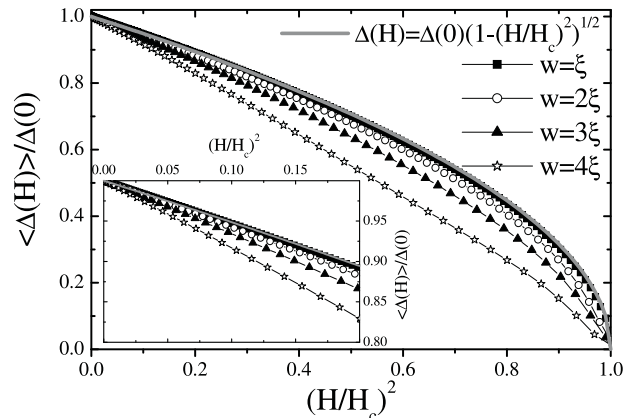


FIG. 4: Dependence of the order parameter averaged over the width of the film on the applied magnetic field for different values of  $w$  (the result is obtained in framework of Ginzburg-Landau equations). It shows a relatively stronger suppression of  $|\Delta|$  in films with  $w > 2\xi$  than in narrow films  $w < 2\xi$  for the same ratio  $H/H_c$ .

The maximal enhancement of the current  $I_{c1}$  does not depend on  $w, d$  (while  $w, d \lesssim \xi$ ) in the second mechanism. The invariance is connected with a universal dependence of the order parameter on a magnetic field  $\Delta(H) = \Delta(0)(1 - (H/H_c)^2)^{1/2}$  for narrow samples. For wider samples  $w > \xi$  the order parameter decreases faster with the growing ratio of  $H/H_c$  (see Fig. 4) and the enhancement of  $I_{c1}$  becomes weaker. For very wide samples the vortices enter the sample at  $H \ll H_c \sim H_{c2}$  and that suppresses the order parameter even stronger. The conclusion is that the second mechanism of enhancement of  $I_{c1}$  is maximal in narrow quasi-1D samples with  $w, d \lesssim \xi$ .

We can speculate that the observed in Ref. [1] suppression of the NMR for wide bridges is connected with exceeding width of the sample over the coherence length. For example the minimal NMR occurs for a sample with  $w = 100nm$  (see Fig. 4 of Ref. [1]). Taking into account the coherence length of bulk lead ( $\xi(0) = 83nm$ ) and a dirty limit expression for the coherence length  $\xi \sim \sqrt{\hbar D / |\Delta|}$  we have  $\xi(T = 1.6K) \simeq 50nm$  (for samples with  $T_c \simeq 2.5K$ ). Therefore, for the widest sample our rough estimation gives  $w \simeq 2\xi$ . Besides from our theory of the second mechanism of NMR it follows that the maximal current enhancement (or maximal suppression

sion of the resistance) occurs at  $H^* \sim H_c \sim 1/(w, d)$  for  $w, d \lesssim \xi$  (see Fig. 3a). In Refs. [1, 4] qualitatively the same dependencies were observed.

Our both mechanisms give a suppression of NMR at approaching  $T_c$  (similar to experiments [1, 2, 3, 4, 5, 6]). The first mechanism becomes noticeable if the length of the wire is not very short. Otherwise the current  $I_{c1}$  is equal to  $I_{c2}$  and  $I_{c2}$  decreases for wires with  $L \lesssim 4\xi(T)$  due to a strong proximity effect from the normal banks.

The second mechanism of NMR is effective only when the term  $\gamma(0)(H/H_c(0))^2$  is larger than the unity (see Eq. (7)). It is obvious that for some temperatures close to  $T_c$  the above condition fails because  $H < H_c(T) \ll H_c(0)$ . Besides with increasing temperature the charge imbalance length increases as  $\lambda_Q \sim \lambda_Q(0)(1 - T/T_c)^{-1/2}$  [7] and the length of the sample becomes smaller than  $\lambda_Q$ . It also suppresses the second mechanism of NMR (see Fig. 3b).

The proposed mechanisms of the negative magnetoresistance are different from those studied in Refs. [23, 24]. The authors of work [23] supposed that *additional* resistance due to NS boundaries stabilizes the superconducting phase. In our approach the intrinsic dissipation grows due to *decreased* intrinsic resistance of the phase slip process and it suppresses the fluctuations in the system.

In Ref. [24] a new channel of dissipation in superconducting wires was proposed which can be suppressed by an external magnetic field. It would be interesting to compare the contributions of that channel and the second mechanism of NMR studied in the present paper.

Our mechanisms of current enhancement are also

rather different from a critical current enhancement predicted in Refs. [26, 27, 28]. In those works the enhancement of current  $I_{c2}$  (using our terminology) was found due to *suppression* by applied magnetic field the pair-breaking resulting from total 'spin-flip'+'non-spin-flip' rate [28]. This process leads to decreasing the current  $I_{c1}$  because  $\lambda_Q$  grows according to that effect. In our second mechanism the increased pair-breaking (due to orbital effect) *decreases* current  $I_{c2}$  but *enhances* the current  $I_{c1}$ .

Our first mechanism has rather different behavior on magnetic field and cannot be confused with theory of Refs.[27, 28]. The direct way to distinguish among the second mechanism of a current enhancement is predicted in our paper and Refs. [27, 28] is to study the samples of different lengths and widths (with other close parameters). In contrast to Refs. [27, 28] the current enhancement in our second mechanism depends on the length of a nanowire and becomes weaker for wires with the length  $L < 2\lambda_Q$ . The second difference is that the maximal current enhancement does not depend on the width of a wire for narrow samples  $w \lesssim \xi$  while in [26, 27] the strong dependence on  $w$  was predicted even for such a narrow wires.

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